Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 1

Exercise 1

Given the function

$f(x,y) = -x^3 + y^3$ defined over the domain $D \equiv$
$3x^2 + 6y^2 \leqslant 108$, compute its absolute maxima and minima.

1) The value of the minimum is ****. 5 ****

2) The value of the minimum is ****. 2 ****

3) The value of the minimum is ****. 0 ****

4) The value of the minimum is ****. 9 ****

5) The value of the minimum is ****. 8 ****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $(-4 + x)^2 + \frac{1}{4}(-2 + y)^2 \leq 1$

1) 317.625

2) 66.7588

3) 276.738

4) 4.294

5) 42.8065

Exercise 3

Compute the mean curvature for $X(u,v) = \{u, 2u^2, v\}$ at the point $(u,v) = (1,3)$.

1) $H(1,3) = 6.78202$

2) $H(1,3) = 7.7564$

3) $H(1,3) = 6.58852$

4) $H(1,3) = 8.06062$

5) $H(1,3) = 0.0285336$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, y)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + uv, 2 + 2uv, v^2)$

Compute $\int_X F$.

1) −9. 2) −1.6 3) 0.2 4) −3.4

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t->\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 7\sin(5x) - 9\sin(x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.003 , by separation of variables.

1) u(2 , 0.003) = *6*.****

2) u(2 , 0.003) = *1*.****

3) u(2 , 0.003) = *5*.****

4) u(2 , 0.003) = *4*.****

5) u(2 , 0.003) = *8*.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 2

Exercise 1

Given the function

$f(x,y) = 5x^3 - 5y^3$ defined over the domain D≡ $30x^2 + 45y^2 \leqslant 2100$, compute its absolute maxima and minima.

1) The value of the minimum is ****.7****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.3****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $(-5+x)^2 + \frac{1}{4}(-2+y)^2 \leq 1$

1) 1336.37

2) 2587.36

3) 340.889

4) 570.199

5) 1757.97

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (4, 2)$.

1) H (4 , 2) = 1.53035

2) H (4 , 2) = -8.81327

3) H (4 , 2) = 0.176777

4) H (4 , 2) = 7.24505

5) H (4 , 2) = 4.68635

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u, u + v^2, uv)$

Compute $\int_X F$.

1) -0.0833333 2) -1.28333 3) -1.78333 4) -0.383333

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0,\ \mathrm{Lim}_{t\to\infty} u(x,t)=0 & 0\le t \\ u(x,0)=x & 0\le x\le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.009 , by separation of variables by means of a Fourier series of order 2.

1) u(1 , 0.009) = ***.5***

2) u(1 , 0.009) = ***.1***

3) u(1 , 0.009) = ***.7***

4) u(1 , 0.009) = ***.3***

5) u(1 , 0.009) = ***.6***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 3

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-2x^3 - 2y^3}{9x + 3x^2 + x^3 - 6x^4 + 3x^5 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -29

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -56

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \leq 49,\ z \geq 7\left(x^2 + y^2\right)\right\}$

1) 4.35365

2) -9.79572

3) 3.26524

4) 10.8841

5) 3.26524

Exercise 3

Compute the center of curvature for $C(t) = \left\{t^2,\ 3 + 2t,\ -3t^2\right\}$ at the point $t = 1$.

1) {18.5, -8.5, -23.2}

2) {6.6, -1.7, 2.3}

3) {1.5, -6.8, -6.2}

4) {11.7, -15.3, -6.2}

5) {3.2, -17., -9.6}

Exercise 4

Consider the vectorial field $F(x, y) = ($
$y\,(3y + 1)\cos(x y) - 2x\,,\ 3\sin(x y) + x\,(3y + 1)\cos(x y) + x$
$)$. Compute the potential function for this field whose potential at the origin is 1 .

. Calculate the value of the potential at the point $p = (-5, -2)$.

1) -32.2799 2) 13.9201 3) -56.4799 4) -21.2799

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -5\cos(3x) - 9\cos(5x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.008$.

1) $u(2, 0.008) =$ **0.****

2) $u(2, 0.008) =$ **6.****

3) $u(2, 0.008) =$ **5.****

4) $u(2, 0.008) =$ **7.****

5) $u(2, 0.008) =$ **1.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 4

Exercise 1

Given the functions

$f(x, y) = (-1 + 2x - 3x^2 - 2y - xy - 3y^2, -3 + 3x + x^2 + y + 3xy + y^2)$

and

$g(u, v) = (3 - 2u - 3u^2 - v - 2uv + v^2, -3u + 3u^2 - 3v - 3uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -1)$.

1) $-396.$

2) -602.747

3) -559.534

4) -519.295

5) -549.063

Exercise 2

Compute $\int_D (2y)\,dx\,dy$ for $D = \{0 \le -5x \le 4,\ 0 \le 9x + 7y \le 4\}$

1) 0.131429

2) 0.831429

3) -1.16857

4) 0.0314286

5) 0.731429

Exercise 3

Compute the center of curvature for $C(t) = \{-2t^2, t^2, 3 - 2t\}$ at the point $t = 1$.

1) $\{-1.6, 9.9, 1.3\}$

2) $\{-9.4, -0.5, 10.4\}$

3) $\{-1.6, -4.4, 20.8\}$

4) $\{-6.8, 3.4, 13.\}$

5) $\{-9.4, 12.5, 14.3\}$

Exercise 4

Consider the vector field $F(x,y,z)=(0,-x,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,2,2t^2)$

Compute $\int_\sigma F.$

1) -0.6 2) -3.8 3) 0. 4) -2.5

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=9(6-6t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=-5\sin(x)-7\sin(3x) & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.001$, by separation of variables.

1) u(2 , 0.001) = **1.****

2) u(2 , 0.001) = **3.****

3) u(2 , 0.001) = **8.****

4) u(2 , 0.001) = **0.****

5) u(2 , 0.001) = **7.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 5

Exercise 1

Given the functions

$f(x, y) = (3x^2 - 3y + 3xy - 2y^2, 2 + 2x + 2x^2 + 2y - xy + 3y^2)$

and

$g(u, v) = (1 + 2u^2 + v + 2uv + 2v^2, -3 + u - 2u^2 - v - 2uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, 1)$.

1) 18 506.3

2) 36 271.

3) 49 621.8

4) 57 748.5

5) 49 450.4

Exercise 2

Compute $\int_D (2x + y)\, dx dy$ for $D = \{0 \le 3x - 7y \le 5, 0 \le 5x + 2y \le 3\}$

1) 0.205235

2) 0.505235

3) −1.19477

4) 0.305235

5) 1.30523

Exercise 3

Compute the center of curvature for $C(t) = \{3t^2, -t^2, 2 + 2t\}$ at the point $t = -3$.

1) {27.4, −81.4, 650.4}

2) {190., 135.4, 162.6}

3) {27.4, −244., 704.6}

4) {81.6, −27.2, 542.}

5) {−406.2, 27., 650.4}

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (3, 2t, 2t^2)$

Compute $\int_\sigma F$.

1) 10.8 2) -6. 3) -13.2 4) -28.2

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi,\ 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$
and the moment $t = 0.008$ by means of a Fourier series of order 2 .

1) u(2 , 0.008) = **9.****

2) u(2 , 0.008) = **3.****

3) u(2 , 0.008) = **6.****

4) u(2 , 0.008) = **2.****

5) u(2 , 0.008) = **5.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 6

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - 2y^4}{-2x - x^2 + x^4 - x^5 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -15

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -31

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 25,\ z \geq 15(x^2 + y^2)\}$

1) 3.12074

2) 7.28173

3) 1.04025

4) 2.60062

5) 7.02167

Exercise 3

Compute the center of curvature for $C(t) = \{2t^2, -2 - 3t, t^2\}$ at the point $t = 0$.

1) {0.4, -2.4, -0.7}

2) {2.6, -3.2, 2.3}

3) {0.4, -2.6, 1.5}

4) {1., -1., 1.1}

5) {1.8, -2., 0.9}

Exercise 4

Consider the vectorial field $F(x, y) = ($
$-y(3y - 2)\sin(xy) - 3,\ -(x(3y - 2)\sin(xy)) + 3\cos(xy) + x$
$)$. Compute the potential function for this field whose potential at the origin is -7.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) -15.8131 2) -25.4131 3) -7.01307 4) -10.2131

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(5,t) = 0 & 0 \le t \\ u(x,0) = 2\cos\left(\frac{\pi x}{5}\right) - 9\cos(\pi x) & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=3$ and the moment $t=0.003$.

1) u(3 , 0.003) = **2.****

2) u(3 , 0.003) = **7.****

3) u(3 , 0.003) = **0.****

4) u(3 , 0.003) = **8.****

5) u(3 , 0.003) = **9.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 7

Exercise 1

Given the function

$f(x,y) = 5x^3 + 4y^3$ defined over the domain D≡ $45x^2 + 24y^2 \leqslant 2004$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.9****

5) The value of the minimum is ****.8****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $\frac{1}{16}(-5+x)^2 + \frac{1}{9}(-5+y)^2 \leq 1$

1) 5042.81

2) 600.63

3) 1060.29

4) 3505.84

5) 3540.85

Exercise 3

Compute the mean curvature for $X(u,v) = \{Cos[u], Sin[u], v\}$ at the point $(u,v) = (6, 10)$.

1) $H(6,10) = -2.73233$

2) $H(6,10) = -7.82263$

3) $H(6,10) = 0.5$

4) $H(6,10) = 3.63396$

5) $H(6,10) = 7.78125$

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, 0)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u + 2v, u^2, 2)$

Compute $\int_X F$.

1) 2.1 2) -0.5 3) 0. 4) 1.6

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0 & 0 \le t \\ u(x,0)=6\sin\left(\frac{\pi x}{4}\right) & 0 \le x \le 4 \\ \frac{\partial}{\partial t}u(x,0)=x & 0. \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 3
and the moment t= 0.001 by means of a Fourier series of order 1 .

1) u(3 , 0.001) = **3.****

2) u(3 , 0.001) = **1.****

3) u(3 , 0.001) = **0.****

4) u(3 , 0.001) = **5.****

5) u(3 , 0.001) = **4.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 8

Exercise 1

Given the function

$f(x,y) = x^3 - 5y^3$ defined over the domain $D \equiv 6x^2 + 15y^2 \leqslant 156$, compute its absolute maxima and minima.

1) The value of the maximum is ****.0****

2) The value of the maximum is ****.4****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.6****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $\frac{1}{9}(-3 + x)^2 + (-3 + y)^2 \leq 1$

1) 1090.82

2) 2185.87

3) 579.624

4) 51.8596

5) 134.724

Exercise 3

Compute the mean curvature for $X(u,v) = \left\{v^2 \operatorname{Cos}[u], v^2 \operatorname{Sin}[u], v\right\}$ at the point $(u,v) = (2, 7)$.

1) H (2 , 7) = 3.03356

2) H (2 , 7) = -2.2197

3) H (2 , 7) = 6.42269

4) H (2 , 7) = 5.18361

5) H (2 , 7) = 0.000365351

Exercise 4

Consider the vector field $F(x,y,z) = (x, z, 0)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v^2, 1 + uv, 1 + 2v^2)$

Compute $\int_X F$.

1) 2.   2) 1.6   3) -2.3   4) 2.1

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 6\sin(6x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = 8\sin(6x) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.007$.

1) u (1 , 0.007) = **7.****

2) u (1 , 0.007) = **2.****

3) u (1 , 0.007) = **1.****

4) u (1 , 0.007) = **5.****

5) u (1 , 0.007) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 9

Exercise 1

Given the functions

$f(x, y) = (3 - 3x^2 + 2y + 3xy, 3x - xy + 3y^2)$

and

$g(u, v) = (-1 - 3u + 2u^2 + 2v - 3uv + v^2, 3 + u + u^2 + uv - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, 0)$.

1) −582.631

2) −253.984

3) −651.902

4) −711.678

5) −456.

Exercise 2

Compute $\int_D (4x)\,dx\,dy$ for $D = \{0 \le -3x - 3y \le 5, 0 \le 7x - 8y \le 2\}$

1) 1.4642

2) −1.6358

3) −2.3358

4) −0.335802

5) 1.0642

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (5, 8)$.

1) $H(5,8) = -4.14621$

2) $H(5,8) = 0.5$

3) $H(5,8) = -5.70228$

4) $H(5,8) = -8.27556$

5) $H(5,8) = -3.74823$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, y, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t^2, 1 + t^2, 2t)$

Compute $\int_\sigma F$.

1) −22.1667 2) 9.83333 3) 4.83333 4) −4.16667

Exercise 5

$$\begin{cases} (8 + 4x)(1+t+t^2)\frac{\partial u}{\partial t}(x,t) = (1+8x+2x^2)(1+2t)\frac{\partial u}{\partial x}(x,t) & 0<x<4,\ 0<t \\ u(x,0) = 3\left(2x^2+8x+1\right)^3 - \frac{3}{\left(2x^2+8x+1\right)^2} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary proble at the points x= 3 , t= 0.001 , by means of the technique of separation of variables.

1) u(3 , 0.001) = 1*****.****

2) u(3 , 0.001) = 2*****.****

3) u(3 , 0.001) = 3*****.****

4) u(3 , 0.001) = 8*****.****

5) u(3 , 0.001) = 5*****.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 10

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-3x^4 - y^4}{6x + 4x^2 + x^4 - 2x^5 - 2x^6 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 58

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -84

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 64,\ z \ge 13(x^2 + y^2)\}$

1) 4.61765

2) 4.61765

3) 2.30883

4) 7.69609

5) -3.84805

Exercise 3

Compute the center of curvature for $C(t) = \{-3t^2, -2 + 3t, -t^2\}$ at the point $t = -3$.

1) {25.05, 322.2, 44.15}

2) {-82.35, 358., -27.45}

3) {-332.95, 250.6, 294.75}

4) {-368.75, 286.4, -349.65}

5) {-46.55, 680.2, 187.35}

Exercise 4

Consider the vectorial field $F(x, y) = (2x(2y+1)y^2 + 2x,\ 2x^2y^2 + 2x^2y(2y+1))$. Compute the potential function for this field whose potential at the origin is 5.

. Calculate the value of the potential at the point $p = (-7, 2)$.

1) 3515.6 2) -2895.2 3) 1551. 4) 1034.

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = -7\cos\left(\frac{\pi x}{4}\right) - 4\cos(\pi x) & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 3$ and the moment $t = 0.001$.

1) u(3 , 0.001) = **8.****

2) u(3 , 0.001) = **9.****

3) u(3 , 0.001) = **0.****

4) u(3 , 0.001) = **5.****

5) u(3 , 0.001) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 11

Exercise 1

Given the function

$f(x,y) = 5x^3 + 4y^3$ defined over the domain D≡ $15x^2 + 12y^2 \leq 108$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.1****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $\frac{1}{25}(-5+x)^2 + (-5+y)^2 \leq 1$

1) 1228.31

2) 1779.73

3) 443.75

4) 200.316

5) 2153.87

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (1, 5)$.

1) $H(1,5) = -4.23439$

2) $H(1,5) = -2.54958$

3) $H(1,5) = 0.5$

4) $H(1,5) = 5.61438$

5) $H(1,5) = -6.29419$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, x, 0)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + uv, 2, 2u^2)$

Compute $\int_X F$.

1) 2.66667 2) 6.46667 3) 2.46667 4) 0.466667

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = 8\sin\left(\frac{\pi x}{4}\right) & 0 \le x \le 4 \\ \frac{\partial}{\partial t}u(x,0) = x & 0. \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
 and the moment t= 0.005 by means of a Fourier series of order 1 .

1) u(1 , 0.005) = **8.****

2) u(1 , 0.005) = **0.****

3) u(1 , 0.005) = **1.****

4) u(1 , 0.005) = **5.****

5) u(1 , 0.005) = **3.****

Further Mathematics - Degree in Engineering - 2025/2026
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Exercise 1

Given the functions

$f(x, y) = (3x^2 + 3y + 3y^2, x + y + xy + 2y^2)$

and

$g(u, v) = (1 + 3u + 2u^2 + 2v + 2uv + v^2, u + 3u^2 + v - 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -1)$.

1) 3707.12

2) 5234.03

3) 2892.

4) 3197.53

5) 5307.64

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \le 6x + 6y \le 2, 0 \le 6x - 2y \le 4\}$

1) 0.255556

2) 1.25556

3) 1.25556

4) 0.0555556

5) −1.64444

Exercise 3

Compute the center of curvature for $C(t) = \{-3t^2, -t^2, -1 + t\}$ at the point $t = 3$.

1) {567.45, 945.85, −108.1}

2) {−81.15, −27.05, −1081.}

3) {567.45, −675.65, −1945.8}

4) {891.75, −783.75, −216.2}

5) {243.15, −999.95, −756.7}

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + 2t, 2, 2t + 2t^2)$

Compute $\int_\sigma F.$

1) 1.16667   2) 2.76667   3) 0.666667   4) 4.16667

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.003$ by means of a Fourier series of order 2 .

1) $u(2, 0.003)$ = $**8.****$

2) $u(2, 0.003)$ = $**5.****$

3) $u(2, 0.003)$ = $**4.****$

4) $u(2, 0.003)$ = $**6.****$

5) $u(2, 0.003)$ = $**2.****$

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 13

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-3x^4 - 3y^4}{6x + 9x^2 + x^4 + 6x^5 - 3x^6 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -51

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 112

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2 \le 121,\ z \ge 4\,(x^2+y^2)\}$

1) 117.452

2) −42.2826

3) −9.39614

4) 46.9807

5) −18.7923

Exercise 3

Compute the center of curvature for $C(t)=\{-3t^2, -3t^2, 3-2t\}$ at the point $t=3$.

1) {−958.833, 308.667, 780.}

2) {−763.833, −958.833, 1267.5}

3) {−81.3333, −81.3333, 975.}

4) {−471.333, −276.333, 1560.}

5) {−666.333, −276.333, 1657.5}

Exercise 4

Consider the vectorial field $F(x,y)=(2xy^2, 2x^2y+x)$. Compute the potential function for this field whose potential at the origin is 7.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) −6.18889   2) 6.41111   3) 1.51111   4) 7.11111

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 8\sin(7x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -9\sin(7x) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.009$.

1) u (1 , 0.009) = **2.****

2) u (1 , 0.009) = **8.****

3) u (1 , 0.009) = **0.****

4) u (1 , 0.009) = **5.****

5) u (1 , 0.009) = **7.****

Further Mathematics - Degree in Engineering - 2025/2026
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Exercise 1

Given the functions

$f(x, y) = (3 + x + x^2 + 3y - 3xy - 3y^2, -2 + x + 3x^2 + 3y - 2xy)$

and

$g(u, v) = (-3 + u - 2u^2 - 3v - 2uv + v^2, 3u - 3u^2 - v + uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, 1)$.

1) 2845.43

2) 2729.12

3) 2117.11

4) 2649.04

5) 1818.

Exercise 2

Compute $\int_D (x + y)\,dx\,dy$ for $D = \{0 \le -5x - 7y \le 3,\ 0 \le -6x + 7y \le 5\}$

1) −0.0619835

2) 1.83802

3) 0.838017

4) −0.461983

5) 0.738017

Exercise 3

Compute the mean curvature for $X(u,v) = \{Cos[u], Sin[u], v\}$ at the point $(u,v) = (6, 2)$.

1) $H(6, 2) = -7.66881$

2) $H(6, 2) = -8.65065$

3) $H(6, 2) = 0.5$

4) $H(6, 2) = -4.62606$

5) $H(6, 2) = 1.65228$

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, -x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + t^2, 1, t^2)$

Compute $\int_\sigma F.$

1) 3.3 2) 2.5 3) 8.7 4) -3.7

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 4\sin(x) + 9\sin(4x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.009$.

1) u(1 , 0.009) = **7.****

2) u(1 , 0.009) = **6.****

3) u(1 , 0.009) = **5.****

4) u(1 , 0.009) = **2.****

5) u(1 , 0.009) = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 15

Exercise 1

Given the function

$f(x,y) = -5x^3 + y^3$ defined over the domain D≡ $45x^2 + 6y^2 \leq 1716$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.6****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.3****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $\frac{1}{4}(-3 + x)^2 + (-2 + y)^2 \leq 1$

1) 268.606

2) 169.31

3) 1255.37

4) 179.794

5) 210.878

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (2, 4)$.

1) H (2 , 4) = -7.79362

2) H (2 , 4) = 0.5

3) H (2 , 4) = 2.50385

4) H (2 , 4) = -8.02075

5) H (2 , 4) = -4.45746

Exercise 4

Consider the vector field $F(x,y,z) = (0, -y, -z)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v^2, uv + 2v^2, u + 2uv)$

Compute $\int_X F$.

1) 2.4 2) -3.2 3) -7.6 4) 12.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -3\sin(2\pi x) & 0 \le x \le 1 \\ \frac{\partial}{\partial t}u(x,0) = x & 0. \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{1}{2}$

and the moment $t = 0.01$ by means of a Fourier series of order 1 .

1) $u\left(\frac{1}{2}, 0.01\right)$ = ***.**1*

2) $u\left(\frac{1}{2}, 0.01\right)$ = ***.**3*

3) $u\left(\frac{1}{2}, 0.01\right)$ = ***.**9*

4) $u\left(\frac{1}{2}, 0.01\right)$ = ***.**2*

5) $u\left(\frac{1}{2}, 0.01\right)$ = ***.**6*

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 16

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-3x^3+y^3}{4x+6x^2+x^3+4x^4-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 5

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 4

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\leq 225,\ z\geq 3(x^2+y^2)\}$

1) 139.812

2) 233.021

3) 116.51

4) −23.3021

5) −11.651

Exercise 3

Compute the center of curvature for $C(t)=\{-2-2t,\ 3t^2,\ 2t^2\}$ at the point $t=-2$.

1) {−63., −5.53846, 3.30769}

2) {−357., 162.462, 66.3077}

3) {−210., 36.4615, 24.3077}

4) {−168., 78.4615, −17.6923}

5) {−315., −5.53846, −143.692}

Exercise 4

Consider the vectorial field $F(x,y)=\left(\dfrac{y(1-xy)}{xy+1}-y\log(xy+1)+3,\ \dfrac{x(1-xy)}{xy+1}-x\log(xy+1)\right)$. Compute the potential function for this field whose potential at the origin is 0.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) −0.380005 2) 4.61999 3) 2.91999 4) 1.61999

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t->\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 7\sin(8x) - 2\sin(5x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.007 , by separation of variables.

1) u(2 , 0.007) = ***.9***

2) u(2 , 0.007) = ***.1***

3) u(2 , 0.007) = ***.4***

4) u(2 , 0.007) = ***.8***

5) u(2 , 0.007) = ***.7***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 17

Exercise 1

Given the function

$f(x,y) = 3x^3 + y^3$ defined over the domain D≡ $18x^2 + 3y^2 \leqslant 300$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.7****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq 3\sqrt{x^2 + y^2}$ and $(-2 + x)^2 + \frac{1}{4}(-2 + y)^2 \leq 1$

1) 261.538

2) 824.803

3) 985.077

4) 146.283

5) 137.405

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \mathrm{Cos}[u], v^2 \mathrm{Sin}[u], v\}$ at the point $(u,v) = (5, 2)$.

1) $H(5,2) = -5.61436$

2) $H(5,2) = -0.521338$

3) $H(5,2) = 0.0160502$

4) $H(5,2) = 5.34517$

5) $H(5,2) = -3.33063$

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, z)$ and the parameterized surface

$X: [0,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + v, u^2 + 2v^2, 2uv)$

Compute $\int_X F$.

1) 0.666667 2) -1.43333 3) 3.46667 4) -3.23333

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -5\sin(2x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = x & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.005 by means of a Fourier series of order 1 .

1) u(1 , 0.005) = ***.**5*

2) u(1 , 0.005) = ***.**1*

3) u(1 , 0.005) = ***.**8*

4) u(1 , 0.005) = ***.**4*

5) u(1 , 0.005) = ***.**7*

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 18

Exercise 1

Given the functions

$f(x, y) = (3 - x + 3y + 2xy - 3y^2, -3 - x - x^2 + xy + 2y^2)$

and

$g(u, v) = (-3 - 2u + u^2 + v - 2uv - v^2, 2 + 3u^2 + 3v - 2uv + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, -1)$.

1) −148.082

2) −40.3732

3) −90.

4) −140.778

5) −76.3932

Exercise 2

Compute $\int_D (3x)\,dx\,dy$ for $D = \{0 \le -8x + 7y \le 9,\ 0 \le x - 5y \le 3\}$

1) −2.85455

2) −4.35455

3) −3.75455

4) −2.45455

5) −4.35455

Exercise 3

Compute the mean curvature for $X(u,v) = \{Cos[u], Sin[u], v\}$ at the point $(u,v) = (6, 2)$.

1) $H(6,2) = 0.5$

2) $H(6,2) = -2.15629$

3) $H(6,2) = -8.37477$

4) $H(6,2) = 1.76767$

5) $H(6,2) = 1.14562$

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, -x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (4, 3t, 1)$

Compute $\int_\sigma F$.

1) 1.4 2) 0. 3) -0.9 4) -3.9

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = 3\sin\left(\frac{\pi x}{3}\right) & 0 \le x \le 3 \\ \frac{\partial}{\partial t}u(x,0) = x^2 & 0. \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 2$
and the moment $t = 0.002$ by means of a Fourier series of order 1.

1) u(2 , 0.002) = **9.****

2) u(2 , 0.002) = **2.****

3) u(2 , 0.002) = **4.****

4) u(2 , 0.002) = **8.****

5) u(2 , 0.002) = **1.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 19

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - y^3}{6x + 6x^2 + x^3 + 2x^4 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -26

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 40

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 16,\ z \ge 9(x^2 + y^2)\}$

1) 2.7541

2) −0.82623

3) 3.02951

4) 2.47869

5) 5.78361

Exercise 3

Compute the center of curvature for $C(t) = \{t^2, t^2, 2 - t\}$ at the point $t = 1$.

1) {8.25, −0.75, 3.}

2) {3.25, 3.25, 10.}

3) {0.25, 0.25, 3.}

4) {7.25, 7.25, 7.}

5) {12.25, 9.25, 11.}

Exercise 4

Consider the vectorial field $F(x, y) = (6y\sin(xy) + 1,\ 6x\sin(xy))$. Compute the potential function for this field whose potential at the origin is -6.

. Calculate the value of the potential at the point $p = (0, -5)$.

1) 11.4   2) 12.   3) −25.8   4) −6.

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = \sin\left(\frac{\pi x}{4}\right) - 6\sin(2\pi x) & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 3$ and the moment $t = 0.003$.

1) u(3 , 0.003) = ***.9***

2) u(3 , 0.003) = ***.0***

3) u(3 , 0.003) = ***.6***

4) u(3 , 0.003) = ***.3***

5) u(3 , 0.003) = ***.7***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 20

Exercise 1

Given the function

$f(x,y) = x^3 - 5y^3$ defined over the domain D≡ $3x^2 + 30y^2 \leqslant 492$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.2****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.6****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $(-4 + x)^2 + \frac{1}{4}(-2 + y)^2 \leq 1$

1) 1193.42

2) 400.553

3) 1234.13

4) 138.028

5) 124.838

Exercise 3

Compute the mean curvature for $X(u,v) = \{u, u^2, v\}$ at the point $(u,v) = (1, 2)$.

1) H(1,2) = -1.85847

2) H(1,2) = 3.71888

3) H(1,2) = 0.0894427

4) H(1,2) = 5.16315

5) H(1,2) = -6.01711

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, y)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (uv, 2v^2, 2 + 2uv)$

Compute $\int_X F$.

1) 13.1 2) -1.9 3) 10.7 4) 3.2

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -5\sin(x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = x^2 & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.007 by means of a Fourier series of order 1 .

1) u(1 , 0.007) = **4.****

2) u(1 , 0.007) = **5.****

3) u(1 , 0.007) = **3.****

4) u(1 , 0.007) = **1.****

5) u(1 , 0.007) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 21

Exercise 1

Given the functions

$f(x, y) = (-x - 2x^2 - 3xy, 2 - x - 3x^2 - 3y - y^2)$

and

$g(u, v) = (-3 + u + 2u^2 - 3v - 2uv + 3v^2, -3u + v + 3uv - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1)$.

1) 12.6719

2) 75.2639

3) 98.6666

4) 68.4215

5) 56.

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq 4x + 7y \leq 1, 0 \leq -3x + 8y \leq 1\}$

1) 0.904984

2) 1.40498

3) 0.104984

4) -1.49502

5) 0.00498398

Exercise 3

Compute the center of curvature for $C(t) = \{-3t^2, -3 - t, 3t^2\}$ at the point $t = 3$.

1) {307.117, 1746.9, -1665.82}

2) {1083.52, 3493.8, -1083.52}

3) {-81.0833, 1941., 81.0833}

4) {1277.62, 1164.6, 469.283}

5) {1665.82, 582.3, 1827.98}

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t + 2t^2, t^2, 1)$

Compute $\int_\sigma F$.

1) -0.5   2) 0.2   3) -4.   4) -0.4

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = x^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$
and the moment $t=0.005$ by means of a Fourier series of order 1 .

1) u(2 , 0.005) = **4.****

2) u(2 , 0.005) = **3.****

3) u(2 , 0.005) = **2.****

4) u(2 , 0.005) = **9.****

5) u(2 , 0.005) = **8.****

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Final Training Exam - January Call - Hand for serial number: 22

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-2x^4-2y^4}{-6x-6x^2+x^4+2x^5+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -164

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -225

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\leq 49,\ z\geq 7\,(x^2+y^2)\}$

1) 27.2103

2) 10.8841

3) 9.79572

4) −5.44207

5) 5.44207

Exercise 3

Compute the center of curvature for $C(t)=\{-2t^2,\ t^2,\ -1+t\}$ at the point $t=0$.

1) {−0.2, 0.1, −1.}

2) {−0.1, −0.3, −0.7}

3) {−0.4, 0.9, −0.7}

4) {−0.3, 0.4, −0.9}

5) {−0.9, −0.4, −1.7}

Exercise 4

Consider the vectorial field $F(x,y)=\left(\dfrac{y\,(3y+1)}{x\,y+1}-2,\ \dfrac{x\,(3y+1)}{x\,y+1}+3\log(x\,y+1)\right)$. Compute the potential function for this field whose potential at the origin is -1. . Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 3.81764 2) 6.21764 3) 6.01764 4) −1.38236

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -8\cos(4x) - 8\cos(6x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.001$.

1) u(1 , 0.001) = **1.****

2) u(1 , 0.001) = **2.****

3) u(1 , 0.001) = **3.****

4) u(1 , 0.001) = **5.****

5) u(1 , 0.001) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 23

Exercise 1

Given the functions

$f(x, y) = (3 + 3x - 3x^2 + y - xy - 3y^2, 3 - 2x + 3x^2 + y - 2xy + 2y^2)$

and

$g(u, v) = (-1 - u - u^2 + 3v + 2uv - 3v^2, -1 - 3u - u^2 + v - uv + 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, -1)$.

1) 77689.1

2) 38793.9

3) 53314.

4) 20146.6

5) 18351.5

Exercise 2

Compute $\int_D (4x)\,dx\,dy$ for $D = \{0 \le -8x + 9y \le 6,\ 0 \le -6x \le 9\}$

1) -3.

2) -4.1

3) -3.6

4) -1.9

5) -1.9

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (4, 2)$.

1) $H(4,2) = 0.5$

2) $H(4,2) = 6.31177$

3) $H(4,2) = 8.64727$

4) $H(4,2) = 2.39006$

5) $H(4,2) = -6.47127$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, 0)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (3t^2, t^2, 2t + 2t^2)$

Compute $\int_\sigma F$.

1) -2.3 2) 3.1 3) 1.5 4) -1.8

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -\sin(5\pi x) - 8\sin(7\pi x) & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{5}$ and the moment $t = 0.003$.

1) $u\left(\frac{3}{5}, 0.003\right)$ = ***.***************2

2) $u\left(\frac{3}{5}, 0.003\right)$ = ***.***************7

3) $u\left(\frac{3}{5}, 0.003\right)$ = ***.***************9

4) $u\left(\frac{3}{5}, 0.003\right)$ = ***.***************3

5) $u\left(\frac{3}{5}, 0.003\right)$ = ***.***************8

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 24

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3+y^3}{-2x-6x^2+x^3+4x^4-4x^5+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 3

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 4

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 225,\ z\geq 8\left(x^2+y^2\right)\right\}$

1) 52.7941

2) 26.397

3) 114.387

4) 43.9951

5) 123.186

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2, -3t, t^2\right\}$ at the point $t=2$.

1) {-73.5, 26.8333, -18.9}

2) {-52.3, 74.5333, -24.2}

3) {-4.6, 85.1333, 28.8}

4) {-25.8, 53.3333, 12.9}

5) {-15.2, 32.1333, 55.3}

Exercise 4

Consider the vectorial field $F(x,y)=\left(\frac{y(1-3xy)}{xy+1}-3y\log(xy+1)+2x,\ \frac{x(1-3xy)}{xy+1}-3x\log(xy+1)\right)$. Compute the potential function for this field whose potential at the origin is -4.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) -8.5242 2) 3.4758 3) -5.3242 4) -3.7242

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0,\ \text{Lim}_{t->\infty} u(x,t)=0 & 0 \le t \\ u(x,0)=3\sin\left(\frac{3\pi x}{4}\right)+4\sin\left(\frac{7\pi x}{4}\right) & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.01 , by separation of variables.

1) u (2 , 0.01) = **3.****

2) u (2 , 0.01) = **5.****

3) u (2 , 0.01) = **2.****

4) u (2 , 0.01) = **8.****

5) u (2 , 0.01) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 25

Exercise 1

Given the functions

$f(x, y) = (-1 + 3xy + 2y^2, 3 + x + 2x^2 + 3xy)$

and

$g(u, v) = (-1 - 3u + 2u^2 - 3v - v^2, 2u + 2u^2 + v - 2uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -1)$.

1) 4635.75

2) 4627.9

3) 2102.95

4) 617.249

5) 2527.

Exercise 2

Compute $\int_D (4x)\,dx\,dy$ for $D = \{0 \le 6y \le 2,\ 0 \le -3x - 7y \le 5\}$

1) −4.41605

2) −2.41605

3) −3.51605

4) −2.71605

5) −1.91605

Exercise 3

Compute the center of curvature for $C(t) = \{3t^2, -3 - 2t, -3t^2\}$ at the point $t = -2$.

1) {−225.567, −116.4, 196.467}

2) {298.233, −203.7, −94.5333}

3) {36.3333, −291., −36.3333}

4) {−196.467, −465.6, −65.4333}

5) {152.733, −87.3, 80.0667}

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, y)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t, 4t, 4)$

Compute $\int_\sigma F$.

1) 3.6 2) -2.8 3) 0. 4) 2.7

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9(4 + 8\,t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = -3\sin\left(\frac{3\pi x}{4}\right) - 9\sin\left(\frac{3\pi x}{2}\right) & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 2$, $t = 0.008$, by separation of variables.

1) u(2, 0.008) = ***.3***

2) u(2, 0.008) = ***.4***

3) u(2, 0.008) = ***.5***

4) u(2, 0.008) = ***.1***

5) u(2, 0.008) = ***.2***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 26

Exercise 1

Given the function

$f(x,y) = -5x^3 - 5y^3$ defined over the domain D≡ $15x^2 + 30y^2 \leq 540$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.2****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.0****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3(x^2 + y^2)$ and $\frac{1}{4}(-2+x)^2 + (-2+y)^2 \leq 1$

1) 684.09

2) 447.017

3) 174.358

4) 252.489

5) 101.935

Exercise 3

Compute the mean curvature for $X(u,v) = \{v \, \mathrm{Cos}[u], v \, \mathrm{Sin}[u], v\}$ at the point $(u,v) = (1, 4)$.

1) H(1,4) = 0.0883883

2) H(1,4) = -7.65512

3) H(1,4) = -7.19218

4) H(1,4) = 1.53678

5) H(1,4) = 8.84095

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, 0)$ and the parameterized surface

$X: [0,1] \times [-1,0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1, 2u^2 + v, 2u)$

Compute $\int_X F$.

1) -3. 2) -5.4 3) 6.4 4) 2.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0,\ \text{Lim}_{t->\infty} u(x,t)=0 & 0\le t \\ u(x,0)=x & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.008 , by separation of variables by means of a Fourier series of order 2.

1) u(1 , 0.008) = ***.7***

2) u(1 , 0.008) = ***.8***

3) u(1 , 0.008) = ***.6***

4) u(1 , 0.008) = ***.2***

5) u(1 , 0.008) = ***.3***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 27

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3+2y^3}{-x-2x^2+x^3+2x^4+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 3

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −4

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 121,\ z\ge 12\,(x^2+y^2)\}$

1) 15.779

2) 25.2464

3) 22.0906

4) 18.9348

5) −3.1558

Exercise 3

Compute the center of curvature for $C(t)=\{-t^2,\ 2t^2,\ -3-2t\}$ at the point $t=3$.

1) {132.8, −158.8, 213.6}

2) {159.5, −25.3, 106.8}

3) {−27.4, 54.8, 267.}

4) {−0.7, −158.8, 453.9}

5) {26., 81.5, 240.3}

Exercise 4

Consider the vectorial field $F(x,y)=(2y\cos(xy)-3,\ 2x\cos(xy)+x)$. Compute the potential function for this field whose potential at the origin is −4 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) −5.02038 2) −2.62038 3) −29.0204 4) −8.02038

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=7\sin\left(\frac{\pi x}{3}\right) & 0\le x\le 3 \\ \frac{\partial}{\partial t}u(x,0)=-9\sin\left(\frac{\pi x}{3}\right) & 0.\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.004$.

1) u (1 , 0.004) = **5.****

2) u (1 , 0.004) = **4.****

3) u (1 , 0.004) = **2.****

4) u (1 , 0.004) = **6.****

5) u (1 , 0.004) = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 28

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{3x^3 - 3y^3}{-4x - 6x^2 + x^3 + 2x^4 + 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -49

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -21

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 100,\ z \ge 8\,(x^2 + y^2)\}$

1) 0.

2) 19.5127

3) 42.928

4) −19.5127

5) 31.2204

Exercise 3

Compute the center of curvature for $C(t) = \{-2t^2,\ 3 + t,\ -t^2\}$ at the point $t = -2$.

1) {−24.2, 163., −12.1}

2) {−56.8, 228.2, −142.5}

3) {8.4, 130.4, 69.4}

4) {−7.9, 244.5, 20.5}

5) {57.3, 179.3, −44.7}

Exercise 4

Consider the vectorial field $F(x, y) = (-4xy^2 - 1,\ -4x^2y)$. Compute the potential function for this field whose potential at the origin is 1.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 1.17778 2) 0.477778 3) 0.277778 4) −0.522222

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 4\sin(\pi x) - \sin\left(\frac{6\pi x}{5}\right) & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.001 , by separation of variables.

1) u(2 , 0.001) = ***.7***

2) u(2 , 0.001) = ***.9***

3) u(2 , 0.001) = ***.8***

4) u(2 , 0.001) = ***.6***

5) u(2 , 0.001) = ***.2***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 29

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3+2y^3}{-3x-2x^2+x^3-2x^4+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 55

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −46

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 144,\ z\ge 9\,(x^2+y^2)\}$

1) 40.0268

2) 47.5318

3) 70.0469

4) 50.0335

5) 25.0167

Exercise 3

Compute the center of curvature for $C(t)=\{1-3t,\ 3t^2,\ -3t^2\}$ at the point $t=-2$.

1) {−57.3, −39.65, 116.05}

2) {−210.1, 170.45, −94.05}

3) {−324.7, 17.65, −132.25}

4) {−191., 36.75, −36.75}

5) {−171.9, 208.65, −55.85}

Exercise 4

Consider the vectorial field $F(x,y)=(-3x^3y^3+3(-3x-3)x^2y^3+6x,\ 3(-3x-3)x^3y^2)$. Compute the potential function for this field whose potential at the origin is −5 .

. Calculate the value of the potential at the point $p=(-5,-1)$.

1) −157.   2) 1570.   3) 2198.   4) 1099.

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0 & 0 \le t \\ u(x,0)=5\sin\left(\frac{3\pi x}{2}\right) & 0 \le x \le 4 \\ \frac{\partial}{\partial t}u(x,0)=\sin\left(\frac{3\pi x}{2}\right) & 0. \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=2$ and the moment $t=0.009$.

1) $u(2, 0.009)$ =

.
****
****
****
***6

2) $u(2, 0.009)$ =

.
****
****
****
***3

3) $u(2, 0.009)$ =

.
****
****
****
***4

4) $u(2, 0.009)$ =

.
****
****
****
***9

5) $u(2, 0.009)$ =

.
****
****
****
***0

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 30

Exercise 1

Given the function

$f(x,y) = 5x^3 + 5y^3$ defined over the domain D≡ $30x^2 + 45y^2 \leqslant 2100$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.6****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $(-5+x)^2 + \frac{1}{4}(-2+y)^2 \leq 1$

1) 570.199

2) 2066.94

3) 1904.73

4) 2044.75

5) 483.127

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u], v\,\mathrm{Sin}[u], v\}$ at the point $(u,v) = (4, 2)$.

1) H (4 , 2) = -4.31958

2) H (4 , 2) = -8.69756

3) H (4 , 2) = 0.176777

4) H (4 , 2) = -4.12038

5) H (4 , 2) = 1.89015

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, -y)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, v, u + v^2)$

Compute $\int_X F$.

1) 1.06667 2) -0.833333 3) -1.53333 4) -4.03333

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t->\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = x^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.001 , by separation of variables by means of a Fourier series of order 1.

1) u(2 , 0.001) = **4.****

2) u(2 , 0.001) = **0.****

3) u(2 , 0.001) = **3.****

4) u(2 , 0.001) = **6.****

5) u(2 , 0.001) = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 31

Exercise 1

Given the function

$f(x,y) = 4x^3 - 2y^3$ defined over the domain D≡ $6x^2 + 6y^2 \leqslant 30$, compute its absolute maxima and minima.

1) The value of the maximum is ****.0****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.7****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $\frac{1}{4}(-2+x)^2 + (-3+y)^2 \leq 1$

1) 22.4495

2) 220.841

3) 16.3303

4) 138.735

5) 44.7677

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 3u, v\}$ at the point $(u,v) = (2, 2)$.

1) H(2,2) = 0.55007

2) H(2,2) = 2.92019

3) H(2,2) = 8.23184

4) H(2,2) = 0

5) H(2,2) = 1.64329

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, -y)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2, v, 1 + uv)$

Compute $\int_X F$.

1) -0.6 2) -1.9 3) 1. 4) 0.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \sin(2x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = x^2 & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$
and the moment $t = 0.006$ by means of a Fourier series of order 1 .

1) u(1 , 0.006) = ***.*7**

2) u(1 , 0.006) = ***.*8**

3) u(1 , 0.006) = ***.*1**

4) u(1 , 0.006) = ***.*9**

5) u(1 , 0.006) = ***.*0**

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 32

Exercise 1

Given the function

$f(x,y) = -3x^3 - 3y^3$ defined over the domain D≡ $27x^2 + 27y^2 \leqslant 1944$, compute its absolute maxima and minima.

1) The value of the maximum is ****. 3 ****

2) The value of the maximum is ****. 5 ****

3) The value of the maximum is ****. 2 ****

4) The value of the maximum is ****. 6 ****

5) The value of the maximum is ****. 8 ****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3(x^2 + y^2)$ and $\frac{1}{9}(-4+x)^2 + (-4+y)^2 \leq 1$

1) 697.52

2) 685.888

3) 975.465

4) 2216.09

5) 401.764

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, u^2, v\}$ at the point $(u,v) = (2, 5)$.

1) H (2 , 5) = 0.0223607

2) H (2 , 5) = 1.8945

3) H (2 , 5) = -6.4139

4) H (2 , 5) = -1.38994

5) H (2 , 5) = 5.16792

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, x)$ and the parameterized surface

$X: [0,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u^2, 2+u, 2v^2)$

Compute $\int_X F$.

1) 3. 2) 0. 3) -3.5 4) 1.9

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0,\ \mathrm{Lim}_{t->\infty} u(x,t)=0 & 0 \le t \\ u(x,0)=x^2 & 0 \le x \le 3 \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.005 , by separation of variables by means of a Fourier series of order 1.

1) u(2 , 0.005) = **2.****

2) u(2 , 0.005) = **6.****

3) u(2 , 0.005) = **5.****

4) u(2 , 0.005) = **7.****

5) u(2 , 0.005) = **9.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 33

Exercise 1

Given the functions

$f(x,y) = (3 - 3x + 3x^2 + 3y + 3xy + y^2, -x - 3x^2 - y + 2xy)$

and

$g(u,v) = (2 + 2u - 3u^2 + 2uv, -3 - 3u + 3u^2 + 3uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -1)$.

1) −12733.7

2) −9504.

3) −12259.7

4) −3944.76

5) −4174.02

Exercise 2

Compute $\int_D (3x + 3y)\,dx\,dy$ for $D = \{0 \le -6y \le 3,\ 0 \le 3x + 4y \le 7\}$

1) −2.625

2) 4.375

3) 11.8125

4) 7.875

5) 3.5

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (6, 10)$.

1) $H(6,10) = 1.73825$

2) $H(6,10) = 6.72009$

3) $H(6,10) = -3.77072$

4) $H(6,10) = 0.5$

5) $H(6,10) = 5.26746$

Exercise 4

Consider the vector field $F(x,y,z) = (-y, z, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + 2t, 2 + 2t^2, 2)$

Compute $\int_\sigma F.$

1) -28.3333 2) -9.33333 3) 17.6667 4) -43.3333

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{5}$

and the moment $t = 0.003$ by means of a Fourier series of order 1 .

1) $u\left(\frac{3}{5}, 0.003\right)$ = ***.9***

2) $u\left(\frac{3}{5}, 0.003\right)$ = ***.7***

3) $u\left(\frac{3}{5}, 0.003\right)$ = ***.8***

4) $u\left(\frac{3}{5}, 0.003\right)$ = ***.3***

5) $u\left(\frac{3}{5}, 0.003\right)$ = ***.4***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 34

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^4+3y^4}{3x+3x^2+x^4+6x^5+3x^6-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 5

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 7

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 1,\ z\ge 12\,(x^2+y^2)\}$

1) 1.52559

2) 0.125595

3) −1.87441

4) 1.02559

5) 0.325595

Exercise 3

Compute the center of curvature for $C(t)=\{t^2,\ 2t^2,\ -1-3t\}$ at the point $t=-3$.

1) {172.7, 182.5, −325.8}

2) {27.9, 55.8, −181.}

3) {100.3, 37.7, −343.9}

4) {−135., −16.6, −36.2}

5) {64.1, −16.6, −72.4}

Exercise 4

Consider the vectorial field $F(x,y)=(-2y\,e^{xy}-2x-3,\ -2x\,e^{xy})$. Compute the potential function for this field whose potential at the origin is -6.

. Calculate the value of the potential at the point $p=(-1,-9)$.

1) −16 208.2 2) −50 247.1 3) 30 797.9 4) −72 939.7

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = 3\sin(3\pi x) + 2\sin(6\pi x) & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{2}{5}$ and the moment $t = 0.003$.

1) $u\left(\frac{2}{5}, 0.003\right)$ = ***.1***

2) $u\left(\frac{2}{5}, 0.003\right)$ = ***.0***

3) $u\left(\frac{2}{5}, 0.003\right)$ = ***.2***

4) $u\left(\frac{2}{5}, 0.003\right)$ = ***.6***

5) $u\left(\frac{2}{5}, 0.003\right)$ = ***.7***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 35

Exercise 1

Given the function

$f(x,y) = 5x^3 + y^3$ defined over the domain D≡ $45x^2 + 3y^2 \leqslant 1632$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.6****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3\left(x^2 + y^2\right)$ and $\frac{1}{9}(-3+x)^2 + (-3+y)^2 \leq 1$

1) 2529.2

2) 579.624

3) 71.4137

4) 1579.49

5) 2371.8

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (2, 7)$.

1) $H(2,7) = 8.79361$

2) $H(2,7) = 0.5$

3) $H(2,7) = 7.81658$

4) $H(2,7) = -7.16515$

5) $H(2,7) = 5.23731$

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, -x)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v^2, 2 + 2v^2, 2u)$

Compute $\int_X F$.

1) 0. 2) 0.7 3) -0.5 4) -1.3

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=9\sin(\pi x) & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=x^2 & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{1}{2}$

and the moment $t=0.003$ by means of a Fourier series of order 1 .

1) $u(\frac{1}{2}, 0.003) = **0.****$

2) $u(\frac{1}{2}, 0.003) = **2.****$

3) $u(\frac{1}{2}, 0.003) = **8.****$

4) $u(\frac{1}{2}, 0.003) = **1.****$

5) $u(\frac{1}{2}, 0.003) = **9.****$

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 36

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-3x^3+3y^3}{-4x-4x^2+x^3+4x^4+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 15

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 21

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 144,\ z\ge 11\,(x^2+y^2)\}$

1) 28.6796

2) 10.2427

3) -14.3398

4) 8.19418

5) 20.4855

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(2,6)$.

1) $H(2,6) = 5.53013$

2) $H(2,6) = 0.5$

3) $H(2,6) = -4.52212$

4) $H(2,6) = -1.52636$

5) $H(2,6) = -7.51324$

Exercise 4

Consider the vectorial field $F(x,y)=\left(\dfrac{y\,(2xy+3)}{xy+1}+2y\log(xy+1)-6x+3,\ \dfrac{x\,(2xy+3)}{xy+1}+2x\log(xy+1)\right)$. Compute the potential function for this field whose potential at the origin is 0 .

. Calculate the value of the potential at the point $p=(4,8)$.

1) 653.666 2) 198.266 3) 455.666 4) -573.934

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} (5-7x)(1+6t+3t^2)\frac{\partial u}{\partial t}(x,t)=(6+6t)\frac{\partial u}{\partial x}(x,t) & 0<x<3,\ 0<t \\ u(x,0)=-3e^{-2\left(5x-\frac{7x^2}{2}\right)}-2e^{\frac{7x^2}{2}-5x} & 0\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary proble at the points x= 1 , t= 0.001 , by means of the technique of separation of variables.

1) u(1 , 0.001) = ***.2***

2) u(1 , 0.001) = ***.5***

3) u(1 , 0.001) = ***.9***

4) u(1 , 0.001) = ***.7***

5) u(1 , 0.001) = ***.8***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 37

Exercise 1

Given the function

$f(x,y) = -5x^3 - 3y^3$ defined over the domain $D \equiv 15x^2 + 27y^2 \leq 1032$, compute its absolute maxima and minima.

1) The value of the maximum is ****. 6 ****

2) The value of the maximum is ****. 1 ****

3) The value of the maximum is ****. 8 ****

4) The value of the maximum is ****. 3 ****

5) The value of the maximum is ****. 4 ****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $\frac{1}{4}(-2+x)^2 + (-3+y)^2 \leq 1$

1) 44.7677

2) 159.996

3) 175.431

4) 193.96

5) 31.9255

Exercise 3

Compute the mean curvature for $X(u,v) = \{Cos[u], Sin[u], v\}$ at the point $(u,v) = (5, 8)$.

1) $H(5,8) = -7.38321$

2) $H(5,8) = 0.5$

3) $H(5,8) = -7.98851$

4) $H(5,8) = -0.270968$

5) $H(5,8) = 8.89666$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, z)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (4, v^2, 2u^2 + uv)$

Compute $\int_X F$.

1) 5.46667   2) -14.3333   3) -5.33333   4) -8.33333

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 6\sin(2x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = x & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$
 and the moment $t = 0.001$ by means of a Fourier series of order 1 .

1) u (1 , 0.001) = ***.**2*

2) u (1 , 0.001) = ***.**6*

3) u (1 , 0.001) = ***.**8*

4) u (1 , 0.001) = ***.**1*

5) u (1 , 0.001) = ***.**9*

Further Mathematics - Degree in Engineering - 2025/2026
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Exercise 1

Given the functions

$f(x, y) = (3x^2 + 2y + 2y^2, 3x^2 - 2y - xy)$

and

$g(u, v) = (3u + 2u^2 + 3v + 2uv + 2v^2, 1 + 2u + 2u^2 + 2v - 2uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 0)$.

1) $-6804.$

2) $-10873.$

3) -4623.28

4) -1617.39

5) -889.937

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \le 5x + 2y \le 5, 0 \le 4x + 4y \le 4\}$

1) -0.6

2) 1.5

3) $0.$

4) 0.3

5) 1.8

Exercise 3

Compute the center of curvature for $C(t) = \{3t^2, 3t^2, 3 + t\}$ at the point $t = 2$.

1) $\{207.983, 551.783, -343.8\}$

2) $\{322.583, 207.983, -744.9\}$

3) $\{36.0833, 36.0833, -573.\}$

4) $\{551.783, -479.617, -687.6\}$

5) $\{437.183, 322.583, -630.3\}$

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, -x)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (3t^2, t + 2t^2, 2 + 2t^2)$

Compute $\int_\sigma F$.

1) -20.4 2) -17.4 3) -6. 4) 15.6

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.007$ by means of a Fourier series of order 2.

1) $u(1, 0.007)$ = ***.5***

2) $u(1, 0.007)$ = ***.3***

3) $u(1, 0.007)$ = ***.9***

4) $u(1, 0.007)$ = ***.6***

5) $u(1, 0.007)$ = ***.7***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 39

Exercise 1

Given the function

$f(x,y) = 5x^3 - 3y^3$ defined over the domain D≡ $45x^2 + 18y^2 \leq 1908$, compute its absolute maxima and minima.

1) The value of the minimum is ****.0****

2) The value of the minimum is ****.3****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.7****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq 2\sqrt{x^2 + y^2}$ and $(-2 + x)^2 + \frac{1}{4}(-3 + y)^2 \leq 1$

1) 507.684

2) 144.9

3) 480.629

4) 179.071

5) 790.485

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u], e^v \text{Sin}[u], v\}$ at the point $(u,v) = (4, 2)$.

1) H(4,2) = −5.76715

2) H(4,2) = −0.953253

3) H(4,2) = 0.000163226

4) H(4,2) = 1.34469

5) H(4,2) = 4.92986

Exercise 4

Consider the vector field $F(x,y,z) = (-z, x, 0)$ and the parameterized surface

$X: [0,1] \times [-1,0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2 + 2u^2, u, v^2)$

Compute $\int_X F$.

1) 6.5 2) −4.3 3) −1.9 4) 1.7

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=x^2 & 0\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.004 by means of a Fourier series of order 1 .

1) u(2 , 0.004) = **8.****

2) u(2 , 0.004) = **0.****

3) u(2 , 0.004) = **2.****

4) u(2 , 0.004) = **3.****

5) u(2 , 0.004) = **4.****

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Exercise 1

Given the functions

$f(x,y) = (3 + x + x^2 + 3y - y^2, 2 + x - x^2 - 2y - xy + y^2)$

and

$g(u,v) = (-2 - 2u - u^2 + 2v + uv + v^2, -3 - 3u^2 + v - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -1)$.

1) −82.7907

2) −254.775

3) −405.833

4) −300.

5) −429.443

Exercise 2

Compute $\int_D (5y)\,dx\,dy$ for $D = \{0 \le -4x - 6y \le 9,\ 0 \le 2x - y \le 9\}$

1) −42.9148

2) −42.8148

3) −44.5148

4) −41.1148

5) −42.7148

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (6, 6)$.

1) $H(6,6) = 7.29856$

2) $H(6,6) = 0.5$

3) $H(6,6) = 2.38772$

4) $H(6,6) = 5.67432$

5) $H(6,6) = 3.5494$

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, x)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + t^2, 2t, 2t + t^2)$

Compute $\int_\sigma F$.

1) 5.3 2) 3.5 3) 1.4 4) 10.7

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 2$, $t = 0.01$, by separation of variables by means of a Fourier series of order 2.

1) u(2 , 0.01) = **1.****

2) u(2 , 0.01) = **2.****

3) u(2 , 0.01) = **3.****

4) u(2 , 0.01) = **4.****

5) u(2 , 0.01) = **6.****

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Exercise 1

Given the functions

$f(x, y) = (-x - 2x^2 + y + xy + 2y^2, 2 + x + 2x^2 - 3y + 2xy)$

and

$g(u, v) = (2 + 3u - 2u^2 - v - 3v^2, 2 + 2u^2 + v - 2uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1)$.

1) -1046.03

2) -286.193

3) -555.

4) -709.313

5) -429.908

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \le 5x + y \le 9, 0 \le -3x + 7y \le 7\}$

1) 9.19695

2) 12.9839

3) -5.40997

4) 5.40997

5) 3.24598

Exercise 3

Compute the center of curvature for $C(t) = \{2 - 3t, -2t^2, t^2\}$ at the point $t = -3$.

1) {-178., -55.8, 27.9}

2) {-124.6, -180.4, -61.1}

3) {-249.2, 68.8, -25.5}

4) {-53.4, -198.2, 116.9}

5) {-320.4, -216., 134.7}

Exercise 4

Consider the vector field $F(x,y,z) = (0, x, -z)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t + t^2, 2t^2, 1)$

Compute $\int_\sigma F$.

1) 4.96667 2) 2.26667 3) 4.56667 4) 1.66667

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(5,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$
and the moment $t = 0.007$ by means of a Fourier series of order 2.

1) $u(1, 0.007)$ = ***.8***

2) $u(1, 0.007)$ = ***.4***

3) $u(1, 0.007)$ = ***.3***

4) $u(1, 0.007)$ = ***.9***

5) $u(1, 0.007)$ = ***.5***

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Exercise 1

Given the functions

$f(x, y) = (-1 + 3x^2 + 2y - 2xy, -2 - x + x^2 - y + 3xy + y^2)$

and

$g(u, v) = (-2 + u - u^2 + 3v + uv - 2v^2, 3 - u - 3u^2 - v - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, 0)$.

1) −94.

2) −172.105

3) −125.739

4) −175.403

5) −29.7458

Exercise 2

Compute $\int_D (x + 2y)\,dx\,dy$ for $D = \{0 \le x - 2y \le 8,\ 0 \le -7x + 2y \le 2\}$

1) −9.45556

2) −6.95556

3) −7.55556

4) −5.65556

5) −6.55556

Exercise 3

Compute the center of curvature for $C(t) = \{-3t^2, -3t^2, -3 + 3t\}$ at the point $t = 1$.

1) {−34.05, −25.95, −8.1}

2) {−25.95, −28.65, −45.9}

3) {14.55, −1.65, −13.5}

4) {−9.75, −9.75, −27.}

5) {−17.85, 14.55, −5.4}

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t^2, 2t + 2t^2, 2t)$

Compute $\int_\sigma F$.

1) -2.66667 2) -8.06667 3) 1.23333 4) -7.76667

Exercise 5

$$\begin{cases} (-3 + 2x)(1+6t+t^2)\frac{\partial u}{\partial t}(x,t) = (6 + 2t)\frac{\partial u}{\partial x}(x,t) & 0<x<\pi,\ 0<t \\ u(x,0) = 2e^{3(x^2-3x)} - 2e^{x^2-3x} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary proble at the points $x = 1$, $t = 0.003$, by means of the technique of separation of variables.

1) u(1, 0.003) = ***.9***

2) u(1, 0.003) = ***.7***

3) u(1, 0.003) = ***.5***

4) u(1, 0.003) = ***.0***

5) u(1, 0.003) = ***.2***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 43

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{-6x-6x^2+x^3-6x^4+3x^5+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 9

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -27

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 196,\ z\ge 14\,(x^2+y^2)\}$

1) 65.8054

2) 46.0638

3) 10.9676

4) −4.38703

5) 21.9351

Exercise 3

Compute the center of curvature for $C(t)=\{t^2,\ -1-3t,\ -2t^2\}$ at the point $t=1$.

1) {3.9, 5.66667, −7.8}

2) {6., 1.46667, −14.1}

3) {0.4, 8.46667, −2.2}

4) {5.3, 2.86667, −5.7}

5) {1.8, 0.0666667, −5.}

Exercise 4

Consider the vectorial field $F(x,y)=($
$-2xy^2\sin(xy)+2y\cos(xy)-2x,\ 2x\cos(xy)-2x^2y\sin(xy)$
$)$. Compute the potential function for this field whose potential at the origin is -4.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 2.90644 2) −8.69356 3) −3.89356 4) 6.90644

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0,\ \text{Lim}_{t\to\infty} u(x,t)=0 & 0 \le t \\ u(x,0)=5\sin(6x) - 6\sin(x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.002 , by separation of variables.

1) u(2 , 0.002) = **1.****

2) u(2 , 0.002) = **4.****

3) u(2 , 0.002) = **3.****

4) u(2 , 0.002) = **0.****

5) u(2 , 0.002) = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 44

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-2x^4+2y^4}{2x+3x^2+x^4+2x^5-2x^6-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 46

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 30

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 36,\ z\ge 5\,(x^2+y^2)\}$

1) 31.1453

2) -5.56166

3) 17.7973

4) 4.44933

5) 11.1233

Exercise 3

Compute the center of curvature for $C(t)=\{2t^2,\ 3+2t,\ -2t^2\}$ at the point $t=-3$.

1) {185., 87., 293.5}

2) {-119.5, 522., 337.}

3) {-32.5, 652.5, -11.}

4) {54.5, 435., -54.5}

5) {-293.5, 826.5, 119.5}

Exercise 4

Consider the vectorial field $F(x,y)=(3x^2(y-1)y^3+4x+y,\ x^3y^3+3x^3(y-1)y^2)$. Compute the potential function for this field whose potential at the origin is 1 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 1.65417   2) 1.45417   3) -0.645833   4) 4.65417

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = 7\cos(6x) - 3\cos(x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.008$.

1) $u(2, 0.008) =$ **8.****

2) $u(2, 0.008) =$ **0.****

3) $u(2, 0.008) =$ **5.****

4) $u(2, 0.008) =$ **2.****

5) $u(2, 0.008) =$ **7.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 45

Exercise 1

Given the function

$f(x,y) = 5x^3 - 3y^3$ defined over the domain D≡ $30x^2 + 27y^2 \leqslant 1452$, compute its absolute maxima and minima.

1) The value of the minimum is ****.7****

2) The value of the minimum is ****.3****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.6****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq 2\sqrt{x^2 + y^2}$ and $\frac{1}{4}(-4+x)^2 + \frac{1}{9}(-4+y)^2 \leq 1$

1) 6204.51

2) 1328.89

3) 6493.94

4) 4073.86

5) 952.085

Exercise 3

Compute the mean curvature for $X(u,v) = \{v \operatorname{Cos}[u], v \operatorname{Sin}[u], v\}$ at the point $(u,v) = (5, 10)$.

1) H(5,10) = -2.73741

2) H(5,10) = 1.85802

3) H(5,10) = 0.0353553

4) H(5,10) = 5.20372

5) H(5,10) = 2.31803

Exercise 4

Consider the vector field $F(x,y,z) = (z, -y, 0)$ and the parameterized surface

$X: [0,1] \times [-1,0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u^2 + v, 2u^2, 3v)$

Compute $\int_X F$.

1) 1.5 2) 4.8 3) -1.8 4) -3.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.003$, by separation of variables by means of a Fourier series of order 2.

1) u(2 , 0.003) = **6.****

2) u(2 , 0.003) = **3.****

3) u(2 , 0.003) = **2.****

4) u(2 , 0.003) = **4.****

5) u(2 , 0.003) = **7.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 46

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{3x^3 - y^3}{3x + 3x^2 + x^3 + x^4 - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -40

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -24

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 81,\ z \ge 2\left(x^2 + y^2\right)\right\}$

1) -30.9412

2) 80.4472

3) 61.8825

4) 123.765

5) 55.6942

Exercise 3

Compute the center of curvature for $C(t) = \left\{-t^2, -2t^2, -3 - 3t\right\}$ at the point $t = 1$.

1) $\{-8.1, -12.7, 2.96667\}$

2) $\{-9.5, -11.3, 1.56667\}$

3) $\{-3.9, -7.8, 3.66667\}$

4) $\{0.3, -2.2, 5.76667\}$

5) $\{0.3, -5., 5.06667\}$

Exercise 4

Consider the vectorial field $F(x, y) = \left(\dfrac{(-2x - 3)\,y}{x\,y + 1} - 2\log(x\,y + 1) - 3,\ \dfrac{(-2x - 3)\,x}{x\,y + 1}\right)$. Compute the potential function for this field whose potential at the origin is 2 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) -3.89887 2) -2.79887 3) 2.00113 4) -0.398873

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -\sin(5x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = 3\sin(5x) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 2 and the moment t= 0.01 .

1) u(2 , 0.01) = ***.0***

2) u(2 , 0.01) = ***.2***

3) u(2 , 0.01) = ***.9***

4) u(2 , 0.01) = ***.6***

5) u(2 , 0.01) = ***.5***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 47

Exercise 1

Given the functions

$f(x, y) = (-1 + x + 3x^2 - 3y - xy + 2y^2, 2 - 2x^2 - xy + y^2)$

and

$g(u, v) = (2u - 3u^2 + uv - 2v^2, 3 + 2u - u^2 + v + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, 0)$.

1) 72.2691

2) 107.444

3) 79.6836

4) 143.624

5) 92.

Exercise 2

Compute $\int_D (x + 2y)\, dx dy$ for $D = \{0 \leq -x + 2y \leq 4, 0 \leq -7x + 4y \leq 8\}$

1) 2.56

2) -8.88178×10^{-16}

3) 3.84

4) 6.4

5) 15.36

Exercise 3

Compute the center of curvature for $C(t) = \{-2 - 2t, -3t^2, 3t^2\}$ at the point $t = 1$.

1) {34., -9.33333, 9.33333}

2) {54.4, 21.2667, 5.93333}

3) {47.6, -39.9333, -4.26667}

4) {6.8, -5.93333, 33.1333}

5) {23.8, 17.8667, 29.7333}

Exercise 4

Consider the vector field $F(x,y,z) = (y, z, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (1, t + t^2, 2t + t^2)$

Compute $\int_\sigma F$.

1) 0.166667 2) 0.966667 3) −0.233333 4) 3.76667

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9(-6 - 8t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = 4\sin(2\pi x) - 7\sin(3\pi x) & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = \frac{7}{10}$, $t = 0.01$, by separation of variables.

1) $u\left(\frac{7}{10}, 0.01\right) = $ 2********************.****

2) $u\left(\frac{7}{10}, 0.01\right) = $ 3********************.****

3) $u\left(\frac{7}{10}, 0.01\right) = $ 4********************.****

4) $u\left(\frac{7}{10}, 0.01\right) = $ 6********************.****

5) $u\left(\frac{7}{10}, 0.01\right) = $ 8********************.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 48

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{9x+3x^2+x^3-3x^4+6x^5-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -14

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 28

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 169,\ z\ge 7(x^2+y^2)\}$

1) 49.0307

2) 0.

3) 101.833

4) −33.9443

5) 37.7159

Exercise 3

Compute the mean curvature for $X(u,v)=\{3u,\ 2u^2,\ v\}$ at the point $(u,v)=(4,1)$.

1) $H(4,1) = -0.841589$

2) $H(4,1) = 5.55282$

3) $H(4,1) = 5.86428$

4) $H(4,1) = 2.48965$

5) $H(4,1) = 0.00139086$

Exercise 4

Consider the vectorial field $F(x,y)=\left(x y^2(-e^{xy})-y e^{xy}-4x,\ x^2 y(-e^{xy})-x e^{xy}+x\right)$. Compute the potential function for this field whose potential at the origin is -2.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) −4.66705 2) 0.132954 3) −3.06705 4) −6.66705

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 9\sin\left(\frac{\pi x}{2}\right) + 5\sin\left(\frac{3\pi x}{2}\right) & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = \frac{6}{5}$, $t = 0.006$, by separation of variables.

1) $u\left(\frac{6}{5}, 0.006\right)$ = **0.****

2) $u\left(\frac{6}{5}, 0.006\right)$ = **7.****

3) $u\left(\frac{6}{5}, 0.006\right)$ = **3.****

4) $u\left(\frac{6}{5}, 0.006\right)$ = **2.****

5) $u\left(\frac{6}{5}, 0.006\right)$ = **5.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 49

Exercise 1

Given the function

$f(x,y) = 5x^3 - y^3$ defined over the domain D≡ $30x^2 + 9y^2 \leq 804$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.4****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.3****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3(x^2 + y^2)$ and $\frac{1}{4}(-2+x)^2 + (-5+y)^2 \leq 1$

1) 2386.03

2) 2097.75

3) 498.45

4) 345.048

5) 570.199

Exercise 3

Compute the mean curvature for $X(u,v) = \{\mathrm{Cos}[u], \mathrm{Sin}[u], v\}$ at the point $(u,v) = (1,6)$.

1) $H(1,6) = -4.64932$

2) $H(1,6) = -2.75729$

3) $H(1,6) = 0.5$

4) $H(1,6) = 3.18488$

5) $H(1,6) = 6.07966$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -y, -z)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, u + u^2, 2v)$

Compute $\int_X F$.

1) -0.8 2) 2.3 3) 2.8 4) 0.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 8\sin(x) & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = x & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 2
and the moment t= 0.007 by means of a Fourier series of order 1 .

1) u(2 , 0.007) = **8.****

2) u(2 , 0.007) = **1.****

3) u(2 , 0.007) = **2.****

4) u(2 , 0.007) = **7.****

5) u(2 , 0.007) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 50

Exercise 1

Given the functions

$f(x, y) = (3 + 3x + 2x^2 - 2y + xy, -3x^2 - 2y + xy + y^2)$

and

$g(u, v) = (u + 2u^2 - v - 3uv - v^2, 2 - u - 3u^2 + 2v + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, -1)$.

1) -14614.3

2) -2136.65

3) $-10350.$

4) -3334.18

5) -17478.4

Exercise 2

Compute $\int_D (6x)\,dx\,dy$ for $D = \{0 \le 9x + y \le 9, 0 \le 8x - 6y \le 3\}$

1) 0.360328

2) 0.360328

3) 1.68153

4) 1.20109

5) 3.12284

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1, 8)$.

1) $H(1, 8) = 4.29967$

2) $H(1, 8) = -3.1367$

3) $H(1, 8) = -3.23173$

4) $H(1, 8) = 0.5$

5) $H(1, 8) = 7.45078$

Exercise 4

Consider the vector field $F(x,y,z) = (-y, 0, x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t + t^2, 3t^2, 2t^2)$

Compute $\int_\sigma F$.

1) 1.16667 2) −2.23333 3) 5.06667 4) 5.16667

Exercise 5

$$\begin{cases} (8 - x)(1+6t+2t^2)\frac{\partial u}{\partial t}(x,t) = (6 + 4t)\frac{\partial u}{\partial x}(x,t) & 0<x<4,\ 0<t \\ u(x,0) = 2\, e^{-2\left(8x-\frac{x^2}{2}\right)} - 3\, e^{3\left(8x-\frac{x^2}{2}\right)} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary proble at the points x= 1, t= 0.002, by means of the technique of separation of variables.

1) u(1, 0.002) = 7**********.****

2) u(1, 0.002) = 2**********.****

3) u(1, 0.002) = 1**********.****

4) u(1, 0.002) = 9**********.****

5) u(1, 0.002) = 8**********.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 51

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{2x^3+y^3}{9x+9x^2+x^3-3x^4-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 29

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −62

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 9,\ z\ge 11\,(x^2+y^2)\}$

1) 1.26592

2) 0.759552

3) 1.13933

4) 2.02547

5) 1.6457

Exercise 3

Compute the center of curvature for $C(t)=\{-2t^2,\ -3+t,\ -2t^2\}$ at the point $t=3$.

1) {−54.125, −867., −54.125}

2) {119.275, −433.5, −400.925}

3) {292.675, −86.7, 379.375}

4) {552.775, −953.7, 119.275}

5) {466.075, −1127.1, 205.975}

Exercise 4

Consider the vectorial field $F(x,y)=(-y\sin(x\,y)-4x,\ -x\sin(x\,y))$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the value of the potential at the point $p=(-10,-10)$.

1) 217.562 2) −196.138 3) −905.338 4) −294.638

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = 5\sin\left(\frac{\pi x}{3}\right) + \sin(\pi x) & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.002 .

1) u(1 , 0.002) = **3.****

2) u(1 , 0.002) = **4.****

3) u(1 , 0.002) = **1.****

4) u(1 , 0.002) = **8.****

5) u(1 , 0.002) = **0.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 52

Exercise 1

Given the function

$f(x,y) = -2x^3 - 2y^3$ defined over the domain D≡ $6x^2 + 6y^2 \leqslant 48$, compute its absolute maxima and minima.

1) The value of the maximum is ****. 9 ****

2) The value of the maximum is ****. 2 ****

3) The value of the maximum is ****. 0 ****

4) The value of the maximum is ****. 7 ****

5) The value of the maximum is ****. 8 ****

Exercise 2

Compute the integral of $f(x,y,z) = z$

in the domain given by $0 \leq z \leq \sqrt{x^2 + y^2}$ and $\frac{1}{9}(-5+x)^2 + \frac{1}{4}(-3+y)^2 \leq 1$

1) 26.9604

2) 511.492

3) 848.311

4) 351.073

5) 1516.67

Exercise 3

Compute the mean curvature for $X(u,v) = \{u, 2u, v\}$ at the point $(u,v) = (2, 8)$.

1) H (2 , 8) = 0

2) H (2 , 8) = -1.73702

3) H (2 , 8) = -5.90872

4) H (2 , 8) = -3.22333

5) H (2 , 8) = -4.28974

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u^2 + v, 3v, 2u^2 + 2v)$

Compute $\int_X F$.

1) 16.2 2) 6. 3) -4.8 4) 20.4

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = 8\sin(\pi x) & 0 \le x \le 2 \\ \frac{\partial}{\partial t}u(x,0) = x & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{3}{2}$

and the moment $t = 0.004$ by means of a Fourier series of order 1 .

1) $u\left(\frac{3}{2}, 0.004\right)$ = ***.**1*

2) $u\left(\frac{3}{2}, 0.004\right)$ = ***.**3*

3) $u\left(\frac{3}{2}, 0.004\right)$ = ***.**5*

4) $u\left(\frac{3}{2}, 0.004\right)$ = ***.**8*

5) $u\left(\frac{3}{2}, 0.004\right)$ = ***.**9*

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Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^4+y^4}{x+x^2+x^4+2x^5+2x^6-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 2

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 10

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 1,\ z\ge 13\,(x^2+y^2)\}$

1) 1.4163

2) -1.8837

3) 0.516301

4) 1.8163

5) 0.116301

Exercise 3

Compute the center of curvature for $C(t)=\{-3t^2,\ t,\ -t^2\}$ at the point $t=1$.

1) {-9.15, -40., -3.05}

2) {-37.15, -60., -31.05}

3) {14.85, -76., -15.05}

4) {-25.15, -64., -19.05}

5) {14.85, -48., 20.95}

Exercise 4

Consider the vectorial field $F(x, y)=(y-y\cos(x y),\ -x\cos(x y))$. Compute the potential function for this field whose potential at the origin is -6.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) -32.1398 2) -6.23981 3) -8.33981 4) -19.5398

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0 \le t \\ u(x,0)=7\sin(2\pi x)+4\sin(5\pi x) & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{1}{5}$ and the moment $t=0.007$.

1) $u(\frac{1}{5}, 0.007)$ = **4.****

2) $u(\frac{1}{5}, 0.007)$ = **9.****

3) $u(\frac{1}{5}, 0.007)$ = **6.****

4) $u(\frac{1}{5}, 0.007)$ = **5.****

5) $u(\frac{1}{5}, 0.007)$ = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 54

Exercise 1

Given the function

$f(x,y) = -2x^3 - 5y^3$ defined over the domain D≡ $6x^2 + 45y^2 \leq 1644$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.9****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3(x^2 + y^2)$ and $\frac{1}{9}(-5+x)^2 + (-3+y)^2 \leq 1$

1) 4668.62

2) 4627.54

3) 1032.01

4) 66.4322

5) 377.131

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \text{Cos}[u], v^2 \text{Sin}[u], v\}$ at the point $(u,v) = (4, 7)$.

1) H(4,7) = 7.11923

2) H(4,7) = 8.84859

3) H(4,7) = 3.20291

4) H(4,7) = 0.000365351

5) H(4,7) = 6.36999

Exercise 4

Consider the vector field $F(x,y,z) = (-x, -y, z)$ and the parameterized surface

$X: [0,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u, u, 2u + 2uv)$

Compute $\int_X F$.

1) 0. 2) 1.4 3) 0.4 4) 0.6

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 4$ and the moment $t = 0.001$ by means of a Fourier series of order 2 .

1) u(4 , 0.001) = **4.****

2) u(4 , 0.001) = **0.****

3) u(4 , 0.001) = **1.****

4) u(4 , 0.001) = **3.****

5) u(4 , 0.001) = **5.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 55

Exercise 1

Given the functions

$f(x, y) = (-2x - x^2 - y + xy - y^2, -2 + 2x - 3x^2 + 2y - 3xy - 3y^2)$

and

$g(u, v) = (1 - u + 3u^2 - 3v + 3uv + v^2, 1 + 3u - 2u^2 + 3v + uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1)$.

1) 1940.75

2) 1805.5

3) 5837.56

4) 3886.

5) 2935.2

Exercise 2

Compute $\int_D (3y)\,dx\,dy$ for $D = \{0 \le 4x - 4y \le 6, 0 \le 8x - 6y \le 2\}$

1) −13.25

2) −11.25

3) −11.55

4) −11.05

5) −10.25

Exercise 3

Compute the center of curvature for $C(t) = \{3t^2, -t^2, -1 - 3t\}$ at the point $t = 3$.

1) {190.05, 8.45, 394.9}

2) {−204.85, −278.75, 574.4}

3) {333.65, 116.15, 502.6}

4) {10.55, −99.25, 71.8}

5) {82.35, −27.45, 359.}

Exercise 4

Consider the vector field $F(x,y,z) = (x, z, 0)$ and the parameterized curve

$\sigma: [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (1 + 2t, 3, 2)$

Compute $\int_\sigma F$.

1) 0. 2) 2.7 3) 1.3 4) -3.1

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.009$ by means of a Fourier series of order 2.

1) u(1, 0.009) = ***.4***

2) u(1, 0.009) = ***.0***

3) u(1, 0.009) = ***.5***

4) u(1, 0.009) = ***.9***

5) u(1, 0.009) = ***.7***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 56

Exercise 1

Given the function

$f(x,y) = -x^3 + 3y^3$ defined over the domain D≡ $9x^2 + 18y^2 \leq 612$, compute its absolute maxima and minima.

1) The value of the minimum is ****. 9 ****

2) The value of the minimum is ****. 7 ****

3) The value of the minimum is ****. 5 ****

4) The value of the minimum is ****. 2 ****

5) The value of the minimum is ****. 8 ****

Exercise 2

Compute the volume of the domain given by $0 \leq z \leq 3(x^2 + y^2)$ and $\frac{1}{4}(-2+x)^2 + (-4+y)^2 \leq 1$

1) 83.3621

2) 506.032

3) 111.328

4) 400.553

5) 1489.72

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \operatorname{Cos}[u],\ v^2 \operatorname{Sin}[u],\ v\}$ at the point $(u,v) = (1, 8)$.

1) H (1 , 8) = -6.13384

2) H (1 , 8) = 0.000244613

3) H (1 , 8) = 5.35192

4) H (1 , 8) = 5.82872

5) H (1 , 8) = 4.87027

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, 0)$ and the parameterized surface

$X: [0,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u^2, uv, 2v^2)$

Compute $\int_X F$.

1) 2.25556   2) -0.444444   3) 2.95556   4) -1.74444

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = 5\cos\left(\frac{4\pi x}{3}\right) - 3\cos\left(\frac{8\pi x}{3}\right) & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.003$.

1) u(2 , 0.003) = **9.****

2) u(2 , 0.003) = **1.****

3) u(2 , 0.003) = **0.****

4) u(2 , 0.003) = **6.****

5) u(2 , 0.003) = **3.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 57

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4-2y^4}{-2x-2x^2+x^4-2x^5-2x^6+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -7

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -33

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\leq 64,\ z\geq 6\,(x^2+y^2)\}$

1) 16.5818

2) −16.5818

3) 9.9491

4) 28.1891

5) 49.7455

Exercise 3

Compute the center of curvature for $C(t)=\{-2t^2,\ -3t,\ t^2\}$ at the point $t=1$.

1) {−8.5, 1.76667, −0.3}

2) {−7.8, 6.66667, 3.9}

3) {−3.6, 10.1667, 0.4}

4) {−7.1, 4.56667, 2.5}

5) {−9.2, 10.1667, 7.4}

Exercise 4

Consider the vectorial field $F(x,y)=(2x(2y+3)y^2+2x+3,\ 2x^2y^2+2x^2y(2y+3))$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 6.33333 2) 12.3333 3) −14.6667 4) −3.26667

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = -7\cos\left(\frac{\pi x}{3}\right) - 7\cos\left(\frac{7\pi x}{3}\right) & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.009$.

1) u(2 , 0.009) = **4.****

2) u(2 , 0.009) = **1.****

3) u(2 , 0.009) = **5.****

4) u(2 , 0.009) = **6.****

5) u(2 , 0.009) = **9.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 58

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{3x+x^2+x^3-x^4-2x^5-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -58

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 28

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\leq 169,\ z\geq 4\,(x^2+y^2)\}$

1) 72.3053

2) 65.7321

3) 6.57321

4) 72.3053

5) -52.5857

Exercise 3

Compute the center of curvature for $C(t)=\{-t^2,\ t^2,\ 1+2t\}$ at the point $t=-3$.

1) {4.7, 17.1, 87.2}

2) {-93.4, -26.5, 185.3}

3) {-28., 28., 109.}

4) {70.1, 82.5, 130.8}

5) {70.1, -70.1, 130.8}

Exercise 4

Consider the vectorial field $F(x, y) = (-12x - 3y, -3x)$. Compute the potential function for this field whose potential at the origin is 3 .

. Calculate the value of the potential at the point $p=(10, -6)$.

1) -542.1 2) $-417.$ 3) -83.4 4) -458.7

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0,\ \text{Lim}_{t->\infty} u(x,t)=0 & 0 \le t \\ u(x,0)=\sin(x) + 3\sin(8x) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.009 , by separation of variables.

1) u(1 , 0.009) = **5.****

2) u(1 , 0.009) = **9.****

3) u(1 , 0.009) = **3.****

4) u(1 , 0.009) = **2.****

5) u(1 , 0.009) = **8.****

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 59

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \dfrac{-3x^4+3y^4}{-2x-x^2+x^4+x^5+x^6+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 24

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 45

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 36,\ z\ge 10\,(x^2+y^2)\}$

1) 5.608

2) 2.804

3) 10.6552

4) 3.3648

5) 11.7768

Exercise 3

Compute the center of curvature for $C(t)=\{t^2,\ 1-3t,\ 2t^2\}$ at the point $t=-3$.

1) {27.9, -179., 55.8}

2) {153.2, -322.2, 127.4}

3) {81.6, -143.2, 199.}

4) {-115.3, -125.3, -51.6}

5) {-133.2, -71.6, 20.}

Exercise 4

Consider the vectorial field $F(x,y)=(2y^2\sin(xy)-1,\ 2xy\sin(xy)-2\cos(xy))$. Compute the potential function for this field whose potential at the origin is 4 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^2$.

1) 8.1806   2) 2.9806   3) 2.5806   4) 0.780605

5) The vector field is not conservative.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(5,t) = 0 & 0 \le t \\ u(x,0) = 5\cos\left(\frac{7\pi x}{5}\right) - 9\cos\left(\frac{8\pi x}{5}\right) & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 4$ and the moment $t = 0.01$.

1) $u(4, 0.01)$ = ***.3***

2) $u(4, 0.01)$ = ***.1***

3) $u(4, 0.01)$ = ***.0***

4) $u(4, 0.01)$ = ***.8***

5) $u(4, 0.01)$ = ***.9***

Further Mathematics - Degree in Engineering - 2025/2026
Final Training Exam - January Call - Hand for serial number: 60

Exercise 1

Given the function

$f(x,y) = 2x^3 + 2y^3$ defined over the domain D≡ $6x^2 + 9y^2 \leqslant 105$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of the domain given by $0 \le z \le 3(x^2 + y^2)$ and $(-5 + x)^2 + \frac{1}{4}(-3 + y)^2 \le 1$

1) 2711.15

2) 2935.82

3) 2837.33

4) 9.77956

5) 664.447

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \mathrm{Cos}[u], v^2 \mathrm{Sin}[u], v\}$ at the point $(u,v) = (4, 5)$.

1) H (4 , 5) = 4.42808

2) H (4 , 5) = -3.3875

3) H (4 , 5) = 8.01924

4) H (4 , 5) = -2.28094

5) H (4 , 5) = 0.00100489

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, x)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, 2v^2, 2v)$

Compute $\int_X F$.

1) 2.   2) -0.8   3) 3.   4) -3.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = x & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.008 by means of a Fourier series of order 2 .

1) u(2 , 0.008) = **9.****

2) u(2 , 0.008) = **7.****

3) u(2 , 0.008) = **5.****

4) u(2 , 0.008) = **3.****

5) u(2 , 0.008) = **2.****